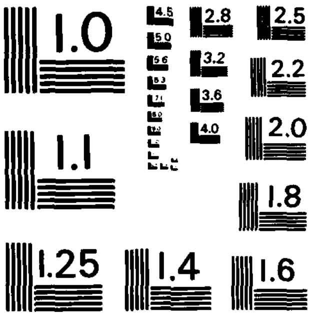
AD-A148 382 MODULATIONAL INSTABILITY OF LOWER HYBRID WAVES(U) REGIS COLL RESEARCH CENTER WESTON MA S P KUO ET AL. 01 JUN 84 AFGL-TR-84-0302 F19628-83-K-0024 1/1

UCLASSIFIED F/G 12/1 NL

END
FILMED
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

UNCLASSIFIED

10

SECURITY CLASSIFICATION OF THIS PAGE

AD-A148 382

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION: Unclassified

1b. RESTRICTIVE MARKINGS: None

2a. SECURITY CLASSIFICATION AUTHORITY: N/A

2b. DECLASSIFICATION/DOWNGRADING SCHEDULE: N/A

3. DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release, distribution unlimited

4. PERFORMING ORGANIZATION REPORT NUMBER(S):

5. MONITORING ORGANIZATION REPORT NUMBER(S): AFGL-TR-84-0302

6a. NAME OF PERFORMING ORGANIZATION: Regis College

6b. OFFICE SYMBOL (If applicable):

7a. NAME OF MONITORING ORGANIZATION: Air Force Geophysics Laboratory/LIS

6c. ADDRESS (City, State and ZIP Code):
235 Wellesley Street
Weston, Mass. 02193

7b. ADDRESS (City, State and ZIP Code):
Hanscom AFB, MA 01731

8a. NAME OF FUNDING/SPONSORING ORGANIZATION: Air Force Geophysics Lab

8b. OFFICE SYMBOL (If applicable): LIS

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: F19628-83-K-0024

8c. ADDRESS (City, State and ZIP Code):
Hanscom AFB, MA 01731
John A. Klobuchar

10. SOURCE OF FUNDING NOS.

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.
62101F	4643	09	AB

11. TITLE (Include Security Classification): Modulational instability of lower hybrid waves

12. PERSONAL AUTHOR(S): S.P. Kuo*, M.C. Lee

13a. TYPE OF REPORT: Scientific - Reprint

13b. TIME COVERED: FROM ______ TO ______

14. DATE OF REPORT (Yr., Mo., Day): June 1, 1984

15. PAGE COUNT: 4

16. SUPPLEMENTARY NOTATION: *Polytechnic Institute of NY, Long Island Center, Farmingdale, NY 11735
Proceedings of 1984 International Conference on Plasma Physics,
Lausanne, Switzerland, June 27 - July 3, 1984

17. COSATI CODES

FIELD	GROUP	SUB. GR.

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number): Modulational instability, lower hybrid waves; Purely growing modes

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

We investigate the excitation of a purely growing modulational mode together with two lower hybrid sidebands by a lower hybrid pump. The condition has been derived. The growth rate of the instability is found to be quite large for the power density employed in the lower hybrid heating experiments in large tokamaks.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐

21. ABSTRACT SECURITY CLASSIFICATION: Unclassified

22a. NAME OF RESPONSIBLE INDIVIDUAL: John A. Klobuchar

22b. TELEPHONE NUMBER (Include Area Code): 617-861-3988

22c. OFFICE SYMBOL: AFGL/LIS

DD FORM 1473, 83 APR — EDITION OF 1 JAN 73 IS OBSOLETE.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE

AFGL-TR-84-0302

Modulational instability of lower hybrid waves

S.P. Kuo

Polytechnic Institute of New York, Long Island Center,
Farmingdale, N.Y. 11735, U.S.A.

M.C. Lee

Regis College Research Center, Weston, Mass. 02193, U.S.A.

Abstract. We investigate the excitation of a purely growing modulational mode together with two lower hybrid sidebands by a lower hybrid pump. The condition, $|k_{oz}/k_{o\perp}| < \omega_{pi}/\Omega_e$, needs to be satisfied for the instability, where $|k_{oz}/k_{o\perp}|$ is the ratio of the perpendicular to the parallel scale lengths of the lower hybrid pump. The growth rate of the instability is found to be quite large for the power densities employed in the lower hybrid heating experiments in large tokamaks.

1. Introduction

The need of supplementary plasma heating in magnetic fusion devices has been generally recognized for achieving the operational temperatures for fusion power production. Among various RF heating schemes, lower hybrid waves have received much attention not only for plasma heating (e.g., Porklab, 1977) but also for plasma confinement (e.g., Fisch, 1978) purposes. The effectiveness of these processes relies on the penetration of RF wave energy into the interior of the plasmas. Hence, the self-modulational effects on the lower hybrid wave propagation are believed to be important on this aspect (e.g., Morales and Lee, 1975). In this paper, we perform a linear stability analyses of the modulational instability whereby a purely growing modulational mode together with two lower hybrid sidebands are excited by a lower hybrid pump wave.

2. Coupled mode equations

We consider the propagation of a lower hybrid pump wave $\phi = \tilde{\phi}_o \exp[i(\underset{\sim}{k}_o \cdot \underset{\sim}{r} - \omega_o t)]$ in a uniform, collisionless plasma embedded in a constant magnetic field $\underset{\sim}{B}_o = \hat{z}B_o$. Here, ϕ_o is the lower hybrid wave field potential; $\underset{\sim}{k}_o (= \hat{y}k_{o\perp} + \hat{z}k_{oz})$ and ω_o are respectively, the wave vector and the angular wave frequency of the lower hybrid pump that satisfies the dispersion relation: $\omega_o = \omega_{LH}[1 + (M/m)(k_{oz}/k_{o\perp})^2]^{1/2}$, where $\omega_{LH} = \omega_{pi}/(1 + \omega_{pe}^2/\Omega_e^2)^{1/2}$ is the lower hybrid resonance frequency.

Proceedings of 1984 International Conference on Plasma Physics, Lausanne, Switzerland, June 27 - July 3, 1984

84 11 27 040

The parameters, M/m, $\omega_{pi}(\omega_{pe})$, and Ω_e are the ratio of ion to electron masses, the ion(electron) plasma frequency, and the electron gyro-frequency, respectively.

The process under consideration is a modulational instability whereby the lower hybrid pump wave ($\omega_o k_o$) excites two lower hybrid side-bands ($\omega_\pm$, $k_\pm$) and a purely growing modulational mode (ω_s ,$\underset{\sim}{k}$). This process can be described by the following frequency and wave vector matching relations: $\omega_+ - \omega_s = \omega_o = \omega_- + \omega_s^*$ and $\underset{\sim}{k}_\pm = \underset{\sim}{k}_o \pm \underset{\sim}{k}$, where $\underset{\sim}{k}$ // $\underset{\sim}{k}_o$. The lower hybrid sidebands are excited through the beating current density driven by the lower hybrid pump field on the density perturbations, $n_s = \tilde{n}_s \exp[i(\underset{\sim}{k}_s \cdot \underset{\sim}{r} - \omega_s t)]$, of the zero-frequency modulational mode.

From the Poisson's equation, $k_\pm^2\, \phi_\pm = 4\pi e(\delta n_{i\pm} - \delta n_{e\pm})$, where the ion and electron density perturbations ($\delta n_{i\pm}$ and $\delta n_{e\pm}$) associated with the lower hybrid sidebands are given, respectively, by $\delta n_{i\pm} = (n_o e/M\omega_o^2)k_\pm^2[\phi_\pm + (k_{o\perp}/k_{\pm\perp})(\omega_o/\omega_\pm)\phi_o(n_{s\pm}/n_o)]$ and $\delta n_{e\pm} = (n_o e/m\Omega_e^2)(1 - k_{oz}^2\Omega_e^2/k_{o\perp}^2\,\omega_o^2)k_{\pm\perp}[\phi_\pm + (k_{o\perp}/k_{\pm\perp})(\omega_o/\omega_\pm)\phi_o(n_{s\pm}/n_o]$, the coupled mode equation for the lower hybrid sidebands is obtained as

$$\phi_\pm = (k_o/k_\pm)(\omega_\pm/\omega_o)(k_{\pm\perp}^2/k_\pm^2)[\omega_{LH}^2\omega_o^2/\omega_{pi}^2\omega_{s\pm}(\omega_{s\pm}\pm 2\omega_o)]\phi_o(n_{s\pm}/n_o) \qquad (1)$$

where the relations $\omega_{s+} = \omega_s = \omega_s^*$ have been used.

The nonlinear effects responsible for the excitation of the purely growing modulational mode include the nonlinear beating current and the pondermotive force appearing , respectively, in the continuity equation and the momentum equation. Under the assumption of quasi-neutrality, the coupled mode equation for the purely growing mode is derived from the continuity and the momentum equations of electrons and ions and has the following form:

$$\begin{aligned}[-\omega_s^2(1 + k_\perp^2 c_s^2/\Omega_e\Omega_i) + k_z^2 c_s^2](n_s/n_o) &= -\omega_s k_\perp(\delta v_{ey}^{NL} + \frac{m}{M}\delta v_{iy}^{NL}) \\ &- {}_s k_z(v_{ez}^{NL} + \frac{m}{M}\delta v_{iz}^{NL}) + k_\perp \frac{\omega_s}{\Omega_e}\left(\frac{F_{ix}}{M} - \frac{F_{ex}}{m}\right) + ik_\perp \frac{\omega_s^2}{m\Omega_e^2}(\\ &F_{iy} + F_{ey}) - i\,\frac{k_z}{M}(F_{iz} + F_{ez}) \end{aligned} \qquad (2)$$

where $C_s = [(T_e + T_i)/M]^{1/2}$ is the ion acoustic velocity. The nonlinear beating current density $n_o \delta \underset{\sim}{v}_{e,i}^{NL}$ and the pondermotive force $\underset{\sim}{F}_{e,i}$ are given, respectively, by

$$n_o \delta \underset{\sim}{v}^{NL}_{e,i} = \delta n_{e,io}\, \delta \underset{\sim}{v}^*_{e,i-} + \delta n^*_{e,io}\, \delta \underset{\sim}{v}_{e,i+} + \delta n^*_{e,i-}\, \delta \underset{\sim}{v}_{e,io} + \delta n_{e,i+} \cdot \delta \underset{\sim}{v}^*_{e,io}$$

and
$$\underset{\sim}{F}_{e,i} = -im_{e,i}\{\underset{\sim}{k}_o \cdot \delta \underset{\sim}{v}^*_{e,i-}\, \delta \underset{\sim}{v}_{e,io} - \underset{\sim}{k}_- \cdot \delta \underset{\sim}{v}_{e,io}\, \delta \underset{\sim}{v}^*_{e,i-} + \underset{\sim}{k}_+ \delta \underset{\sim}{v}^*_{e,io} \cdot \delta \underset{\sim}{v}_{e,i+} - \underset{\sim}{k}_o \cdot \delta \underset{\sim}{v}_{e,i+} \delta \underset{\sim}{v}^*_{e,io}\}$$

where
$$\delta \underset{\sim}{v}_{e,\alpha} = -ik_{\alpha\perp}[\hat{x} + i\hat{y}(\omega_\alpha/\Omega_e) - i\hat{z}(k_{ox}\Omega_e/k_{o\perp}\omega_\alpha)](e\phi_\alpha/m\Omega_e), \quad \alpha = 0 \text{ or } \pm \tag{3}$$

$$\delta \underset{\sim}{v}_{io} = ik_{o\perp}[\hat{x}(\Omega_i/\Omega_o) - i\hat{y} - i\hat{z}(k_{oz}/k_{o\perp})](e\phi_o/M\omega_o), \tag{4}$$

$$\delta \underset{\sim}{v}_{i\pm} = ik_{\pm\perp}[\hat{x}(\Omega_i^2/\omega_\pm^2) - i\hat{y}(\Omega_i/\omega_\pm) - i\hat{z}(k_{oz}\Omega_i/k_{o\perp}\omega_\pm)](e\phi_\pm/m\Omega_e), \tag{5}$$

$$\delta n_{eo} = n_o k_{o\perp}^2(1 - k_{oz}^2\Omega_e^2/k_{o\perp}^2\omega_o^2)(e\phi_o/m\Omega_e^2) \tag{6}$$

$$\delta n_{io} = n_o k_o^2(e\phi_o/M\omega_o^2). \tag{7}$$

With the aid of (3) - (7), two things have been noted from the RHS of (2): Firstly, the ion-nonlinearity terms are relatively insignificant compared to the electron-nonlinearity terms; Secondly, the nonlinear beating current effect is partially cancelled by the pondermotive force effect. Therefore, the electron-nonlinearity term that remains on the RHS of (2) is $-\omega_s kz \delta v^{NL}_{ez}$, contributed from the indiced nonlinear beating current flowing along the imposed magnetic field.

Eliminating n_s and $\phi_\pm$ from Equations (1) and (2) leads to the dispersion relation

$$\omega_s^2 = k_z^2\{C_s^2 - [(1 - k_{oz}^2\Omega_e^2/\omega_{pi}^2 k_{o\perp}^2)\,\Omega_e^2/(1 + Mk_{oz}^2/mk_{o\perp}^2)(\Omega_e^2 + \omega_{pe}^2)] \cdot |v_\phi|^2\} / (1 + k_\perp^2 C_s^2/\Omega_e\Omega_i) \tag{8}$$

where $v_\phi = k_o e\phi_o/m\Omega_e$. For a purely growing mode (i.e., $\omega_s = i\gamma$), the growth rate obtained from (8) is given by

$$\gamma = k_z\{[(1 - k_{oz}^2\Omega_e/\omega_{pi}^2 k_{o\perp}^2)\,\Omega_e^2/(1 + Mk_{oz}^2/mk_{oz}^2)\,(\Omega_e^2 + \omega_{pe}^2)]|v_\phi|^2 - C_s\}^{1/2}/(1 + k_\perp^2 C_s^2/\Omega_e\Omega_i)^{1/2}. \tag{9}$$

Since γ has to be positive for instability to occur, it is clear from (9) that (I) $|k_{oz}/k_{o\perp}| < \omega_{pi}\,\Omega_e$, (10)
namely, the ratio of the perpendicular to the parallel scale lengths of the lower hybrid pump wave has to be less than the ratio of the ion plasma frequency to the electron gyrofrequency, (II) $|\tilde{\phi}_o| > |\tilde{\phi}_{oth}| = ($

$$m/e)(\Omega_e^2 + \omega_{pe}^2)^{1/2}(C_s/k_o)(1 + Mk_{oz}^2/mk_{o\perp}^2)^{1/2}/(1 - \Omega_e^2 k_{oz}^2/\omega_{pi}^2 k_{o\perp}^2)^{1/2} \quad (11)$$

namely, the pump field intensity represented by $|\tilde{\phi}_o|$ has to be greater than a threshold defined by $|\tilde{\phi}_{oth}|$.

The growth rate has a minimum when $k_{oz}^2\Omega_e^2/\omega_{pi}^2 k_{o\perp}^2 = (1 - |\tilde{\phi}_{oth}/\tilde{\phi}_o|^2 / (2 - |\tilde{\phi}_{oth}/\tilde{\phi}_o|^2)$. In this optimum case and for a strong lower hybrid pump wave, (9) may be expressed as

$$\gamma_m \sim k_z |v_\phi| \Omega_e^2/(2\Omega_e^2 + \omega_{pe}^2)^{1/2}(\Omega_e^2 + \omega_{pe}^2)^{1/2}(1 + k_i^2 C_s^2/\Omega_e\Omega_i)^{1/2} \quad (12)$$

which is proporitonal to the wave length of the modulational instability.

4. Summary and conclusions

The condition and the threshold power have been derived for the modulational instability of lower hybrid waves as shown, respectively, in (10) and (11). The nonlinearity for the excitation of purely growing modes is provided by the nonoscillatory beating current in the direction of the imposed magnetic field. For the power densities commonly used in the lower hybrid heating experiments in large tokamaks, the growth rate given in (12) is found to be quite large and it covers a broad spectrum. However, it should be stressed that the spatial modulational on the lower hybrid pump wave as discussed here only occurs in the resonance region where the condition, $|k_{oz}/k_{o\perp}| < \omega_{pi}/\Omega_e$, can be satisfied.

References

Fisch, N.J., Phys. Rev. Lett. 41, 873, 1978.

Morales, G.J., and Y.C. Lee, Phy. Rev. Lett., 35, 930, 1975.

Porklab, M., Phys. Fluids, 20, 2058, 1977.

Acknowledgments. This work was supported jointly in part by the NSF grant ATM-8315322 and in part by the AFSOR grant AFSOF-83-0001 at Polytechnic Institute of New York and by AFGL contract F19628-83-K-0024 at Regis College Research Center.

END

FILMED

1-85

DTIC